111B Section Week 8

Overview: Work on the following problems one at a time, either by yourself or in small-groups. After a sufficient amount of time has passed, we will discuss the solutions as a class. Attending section counts toward your participation grade.

1. Let $I = (2, x)$ be the ideal of $\mathbb{Z}[x]$ generated by 2 and x.

   (a) Show that a polynomial $\sum_{i=0}^{n} a_i x^i \in \mathbb{Z}[x]$ belongs to I if and only if a_0 is even.

   (b) Show that I is a maximal ideal of $\mathbb{Z}[x]$.

2. Let R be a ring and M an ideal.

   (a) Prove that if R/M is a field, then M is a maximal ideal.

   (b) Use 2(a) and the First Isomorphism Theorem to give an alternative proof of 1(b).